

Quantum distortion caused by magneto-elastic coupling for antiferromagnetic tetrahedral cluster of spin-1/2

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We study the effects of magneto-elastic coupling on the degenerate ground spin-state of the antiferromagnetic Heisenberg model on a regular tetrahedron of spin-1/2. When displacement of spin is considered as a classical variable, the degeneracy of the spin-states is lifted through the distance dependence of exchange coupling, i.e., a kind of Jahn-Teller effect takes place. On the other hand, when displacement is considered as a quantum-mechanical variable, the degeneracy of the ground spin-state is not lifted although the tetrahedron is distorted by quantum fluctuation caused by magneto-elastic coupling for one component of the normal coordinates of the doubly degenerate mode. We propose a new model for the structural phase transition of vanadium and nickel spinels: the tetragonal distortion is caused by quantum fluctuation and there exists a kind of hidden order with respect to the non-magnetic ground spin-states.

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I. INTRODUCTION

The highly degenerate antiferromagnetic ground state of the pyrochlore lattice has received considerable attention.¹ It is interesting how the degeneracy is lifted to satisfy the Nernst-Planck theorem.² Reduction of the degeneracy due to lattice distortion through the distance dependence of exchange parameter was investigated before by Terao^{3,4} with classical spins on the basis of the helical spin structure theory.^{5,6} Detailed investigations into the spin Jahn-Teller or Peierls effect were given with classical spin tetramer-model by Tchernyshyov *et al.*^{7,8} and with spin-1/2 tetramer-model for vanadium-spinels by Yamashita and Ueda⁹ in the context of the resonating valence bond approach.¹⁰ In these works, however, the displacement of spin is represented by classical and static variables. Recently, quantum-mechanical treatment of the displacement of spin has given by the present authors¹¹ for a spin-1/2 regular-triangular cluster. They have obtained distortion due to quantum fluctuation caused by the magneto-elastic coupling without lifting the degeneracy, which is a very different aspect from the Jahn-Teller mechanism through lifting the degeneracy of the ground spin-states obtained by the static treatment.

In the present paper, we investigate the effects of the magneto-elastic coupling upon the degenerate ground spin-state of antiferromagnetic (AF) Heisenberg Hamiltonian for a regular tetrahedron of spin-1/2 by treating displacement of spin quantum-mechanically. We propose a new model for the structural phase transition of vanadium and nickel spinels:^{12,13,14} the tetragonal distortion is caused by quantum fluctuation and there exists a hidden spin order with respect to the doubly degenerate non-magnetic ground spin-states.

II. SPIN STATES

The AF Heisenberg Hamiltonian for spins on a regular tetrahedron shown in Fig. 1 is

$$\mathcal{H}_0 = -2J_0 \sum_{\ell < \ell'} \mathbf{s}_\ell \cdot \mathbf{s}_{\ell'}, \quad (1)$$

where $J_0 < 0$ and $s = 1/2$. The symmetry of \mathcal{H}_0 is of the T_d ($\bar{4}3m$) point group. We represent 2^4 spin states by the z components of \mathbf{s}_ℓ and classify them by total S_z as shown in Table I. The $S_z = \pm 2$ states belong to the

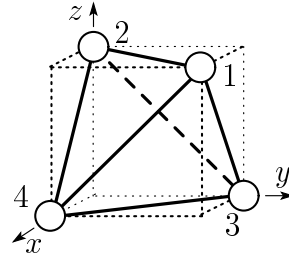


FIG. 1: Labels of spins and orientation of coordinates.

TABLE I: Spin states and S_z .

S_z	State vectors
2	$ ++++>$
1	$ +++->, +-+>, -++>, -+++>$
0	$ ++-->, +-+->, -+->, ---+>, --++>, -+->, --->$
-1	$ ---+>, --++>, -+->, --->, ++-->, +-+->, ++->$
-2	$ ---->$

A_1 representation. Each set of the quadruple $S_z = \pm 1$ states is reduced to $A_1 + T_2$. The hexaple $S_z = 0$ states are reduced to $A_1 + E + T_2$. We write the bases of the

representation as $|A_1, S_z\rangle$ for the singlet A_1 , $|T_2\tau, S_z\rangle$ with $\tau = 1, 2, 3$ for the triplet T_2 , and $|E\eta, S_z\rangle$ with $\eta = u, v$ for the doublet E representations. The eigen-energies are classified by the total spin S as

$$\mathcal{H}_0|A_1, S_z\rangle = -3J_0|A_1, S_z\rangle \quad \text{for } S = 2, \quad (2a)$$

$$\mathcal{H}_0|T_2\tau, S_z\rangle = J_0|T_2\tau, S_z\rangle \quad \text{for } S = 1, \quad (2b)$$

$$\mathcal{H}_0|E\eta\rangle = 3J_0|E\eta\rangle \quad \text{for } S = 0. \quad (2c)$$

For the AF case ($J_0 < 0$), the ground state belongs to the doublet E representation. The spin correlations $\langle E\eta|s_\ell \cdot s_{\ell'}|E\eta\rangle$ for the E spin-states are shown in Fig. 2.

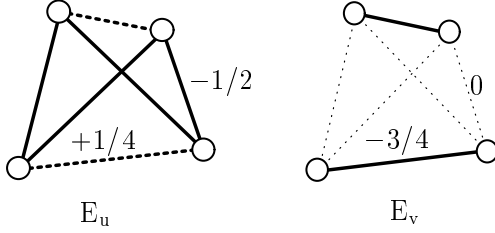


FIG. 2: $\langle E\eta|s_\ell \cdot s_{\ell'}|E\eta\rangle$ for the E spin-states.

III. MAGNETO-ELASTIC COUPLING

Assuming the exchange parameter J depends on the distance between spins, we investigate the effects of the magneto-elastic coupling on the E spin-state. We denote the small deviation of spin around the undistorted position \mathbf{R}_ℓ^0 by \mathbf{u}_ℓ . Normal coordinates of spins are determined by reducing the twelve-dimensional representation by \mathbf{u}_ℓ 's to $A_1 + E + T_1 + 2T_2$. The distortions of the cluster are classified into A_1 (Q_A , its normal coordinate), T_2 (Q_1, Q_2, Q_3) and E (Q_u, Q_v) modes after eliminating the isotropic rotation T_1 and the uniform translation T_2 . The normal modes are written in the twelve-dimensional vector $(\mathbf{u}_1; \mathbf{u}_2; \mathbf{u}_3; \mathbf{u}_4)$ as

$$Q_A(1, 1, 1; -1, -1, 1; -1, 1, -1; 1, -1, -1)/2\sqrt{3} \quad (3)$$

for the singlet A_1 mode,

$$Q_1(0, 1, 1; 0, 1, -1; 0, -1, 1; 0, -1, -1)/2\sqrt{2}, \quad (4a)$$

$$Q_2(1, 0, 1; 1, 0, -1; -1, 0, -1; -1, 0, 1)/2\sqrt{2}, \quad (4b)$$

$$Q_3(1, 1, 0; -1, -1, 0; 1, -1, 0; -1, 1, 0)/2\sqrt{2}, \quad (4c)$$

for the triplet T_2 mode, and

$$Q_u(1, 1, -2; -1, -1, -2; -1, 1, 2; 1, -1, 2)/2\sqrt{6}, \quad (5a)$$

$$Q_v(1, -1, 0; -1, 1, 0; -1, -1, 0; 1, 1, 0)/2\sqrt{2} \quad (5b)$$

for the doublet E mode. The E mode distortion is illustrated in Fig. 3. The bases of the irreducible rep-

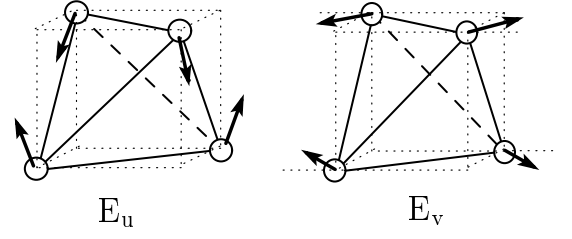


FIG. 3: The E -mode distortion.

resentation made from bilinear combinations of the spin operators are

$$f_A = \sum_{\ell < \ell'} s_\ell \cdot s_{\ell'} / \sqrt{6} \quad (6)$$

for the A_1 representation,

$$f_1 = (s_1 \cdot s_4 - s_2 \cdot s_3) / \sqrt{2}, \quad (7a)$$

$$f_2 = (s_1 \cdot s_3 - s_2 \cdot s_4) / \sqrt{2}, \quad (7b)$$

$$f_3 = (s_1 \cdot s_2 - s_3 \cdot s_4) / \sqrt{2}, \quad (7c)$$

for the T_2 representation, and

$$f_u = [(s_1 + s_2) \cdot (s_3 + s_4) - 2(s_1 \cdot s_2 + s_3 \cdot s_4)] / 2\sqrt{3}, \quad (8a)$$

$$f_v = (s_1 - s_2) \cdot (s_3 - s_4) / 2 \quad (8b)$$

for the E representation.^{7,8,9} The perturbation Hamiltonian is composed of the normal vibrations and couplings of Q_α 's with f_α 's:

$$\begin{aligned} \mathcal{H}' = & \frac{1}{2m}(P_A^2 + P_1^2 + P_2^2 + P_3^2 + P_u^2 + P_v^2) \\ & + \frac{m}{2}[\omega_A^2 Q_A^2 + \omega_T^2(Q_1^2 + Q_2^2 + Q_3^2) \\ & + \omega_E^2(Q_u^2 + Q_v^2)] \\ & - 2[J'_A Q_A f_A + J'_T(Q_1 f_1 + Q_2 f_2 + Q_3 f_3) \\ & + J'_E(Q_u f_u + Q_v f_v)], \end{aligned} \quad (9)$$

where J'_α 's are coupling constants due to the change in J by distortion. When J depends only on the distance between spins, we have

$$J'_A = 2J', \quad J'_T = \sqrt{2}J', \quad J'_E = J'. \quad (10)$$

where $J' = \partial J / \partial a$ and $a = |\mathbf{R}_\ell^0 - \mathbf{R}_{\ell'}^0|$.

In the two dimensional subspace of the ground spin-states, f_α 's are given in the 2×2 matrix form as

$$f_A = -\frac{\sqrt{6}}{4}\sigma_e, \quad (11a)$$

$$f_u = -\frac{\sqrt{3}}{2}\sigma_z, \quad f_v = -\frac{\sqrt{3}}{2}\sigma_x, \quad (11b)$$

$$\text{and } f_\tau = 0, \quad \tau = 1, 2, 3, \quad (11c)$$

where σ_e , σ_z and σ_x are the unit matrix and the Pauli matrices. Moreover, the product representation $T_2 \times E$ is reduced to $T_1 + T_2$, which does not contain the E representation, hence the perturbation belong to the T_2 representation is irrelevant in the subspace of the ground spin-states.

IV. STATIC THEORY

First, we consider the effect of the static displacement of spin. Now, the perturbation is

$$\mathcal{H}' = \frac{m}{2} [\omega_A^2 Q_A^2 + \omega_E^2 (Q_u^2 + Q_v^2)] - 2[J'_A Q_A f_A + J'_E (Q_u f_u + Q_v f_v)]. \quad (12)$$

In the subspace of $|\mathbf{E}\eta\rangle$'s,

$$\mathcal{H}' = \frac{m}{2} [\omega_A^2 Q_A^2 + \omega_E^2 (Q_u^2 + Q_v^2)] + \frac{\sqrt{3}J'_A Q_A}{\sqrt{2}} + \sqrt{3}[J'_E (Q_u \sigma_z + Q_v \sigma_x)], \quad (13)$$

Then the eigenvalues are

$$\delta E' = \frac{m}{2} [\omega_A^2 Q_A^2 + \omega_E^2 (Q_u^2 + Q_v^2)] + \sqrt{\frac{3}{2}} J'_A Q_A \mp \sqrt{3} J'_E \sqrt{Q_u^2 + Q_v^2}. \quad (14)$$

The degeneracy of the ground spin-state is lifted. Minimizing $\delta E'$, we have

$$\delta E'_{\min} = -\frac{3J'^2_A}{4m\omega_A^2} - \frac{3J'^2_E}{2m\omega_E^2}, \quad (15a)$$

$$\text{at } Q_A = -\sqrt{\frac{3}{2}} \frac{J'_A}{m\omega_A^2}, \quad \sqrt{Q_u^2 + Q_v^2} = \frac{\sqrt{3}|J'_E|}{m\omega_E^2}. \quad (15b)$$

The degeneracy of the ground spin-state is lifted by distortion through $Q_u^2 + Q_v^2$ although the ratio of Q_u and Q_v is arbitrary up to quadratic order as shown by Yamashita and Ueda.⁹ The coefficients of the linear combination for the separated spin-state depend on the ratio of Q_u and Q_v , so the perturbed ground state is non-degenerate with respect to the spin state but not unique with respect to the shape of the cluster. Thus a kind of Jahn-Teller mechanism is obtained by the static model although the energy gain due to the distortion is determined by $Q_u^2 + Q_v^2$. These results are compared with the quantum mechanical results in the next section.

V. DYNAMICAL THEORY

Next, we consider the effect of dynamical displacement of spin. By making use of the creation and annihilation

operators, b_α^\dagger and b_α , for normal mode Q_α , we rewrite Eq. (9) as

$$\mathcal{H}' = \sum_\alpha \left[\hbar\omega_\alpha (b_\alpha^\dagger b_\alpha + \frac{1}{2}) - \sqrt{\frac{\hbar}{2m\omega_\alpha}} J'_\alpha (b_\alpha + b_\alpha^\dagger) f_\alpha \right]. \quad (16)$$

Introducing modified operators

$$\tilde{b}_\alpha = b_\alpha - \frac{\sqrt{2}J'_\alpha f_\alpha}{\sqrt{m\hbar\omega_\alpha^3}}, \quad \tilde{b}_\alpha^\dagger = b_\alpha^\dagger - \frac{\sqrt{2}J'_\alpha f_\alpha}{\sqrt{m\hbar\omega_\alpha^3}}, \quad (17)$$

we have

$$\mathcal{H}' = \sum_\alpha \left[\hbar\omega_\alpha (\tilde{b}_\alpha^\dagger \tilde{b}_\alpha + \frac{1}{2}) - \frac{2}{m\omega_\alpha^2} J'^2_\alpha f_\alpha^2 \right]. \quad (18)$$

Commutation relations of the modified operators are

$$[\tilde{b}_\alpha, \tilde{b}_\alpha^\dagger] = 1, \quad [\tilde{b}_\alpha, \tilde{b}_\alpha] = [\tilde{b}_\alpha^\dagger, \tilde{b}_\alpha^\dagger] = 0, \quad (19)$$

so \tilde{b}_α , \tilde{b}_α^\dagger are the Boson operators. Because the commutation relation of f_u and f_v is

$$[f_u, f_v] = i\sqrt{3}/2 \{ (\mathbf{s}_1 - \mathbf{s}_2) \cdot (\mathbf{s}_3 \times \mathbf{s}_4) + (\mathbf{s}_3 - \mathbf{s}_4) \cdot (\mathbf{s}_1 \times \mathbf{s}_2) \}, \quad (20)$$

we have

$$\begin{aligned} [\tilde{b}_u, \tilde{b}_v^\dagger] &= [\tilde{b}_u, \tilde{b}_v] = [\tilde{b}_u^\dagger, \tilde{b}_v^\dagger] \\ &= i \frac{\sqrt{3}J'^2_E}{m\hbar\omega_E^3} \{ [(\mathbf{s}_1 - \mathbf{s}_2) \cdot (\mathbf{s}_3 \times \mathbf{s}_4) + (\mathbf{s}_3 - \mathbf{s}_4) \cdot (\mathbf{s}_1 \times \mathbf{s}_2)] \}, \end{aligned} \quad (21)$$

which characterizes the chirality of the spin tetrahedron. Although the description of the excited states consistent with Eq. (21) are complicated, the ground state with respect to \tilde{b}_α is simply defined as

$$\tilde{b}_\alpha |\Gamma\gamma, S_z > 0\rangle = 0, \quad (22)$$

which is equivalent with

$$b_\alpha |\Gamma\gamma, S_z > 0\rangle = \sqrt{\frac{2}{m\hbar\omega_\alpha^3}} J'_\alpha f_\alpha |\Gamma\gamma, S_z > 0\rangle. \quad (23)$$

In the subspace of the ground spin-states $|\mathbf{E}\eta>_0$'s,

$$\mathcal{H}' = \sum_\alpha \left[-\frac{2}{m\omega_\alpha^2} J'^2_\alpha f_\alpha^2 + \frac{\hbar\omega_\alpha}{2} \right], \quad (24)$$

where f_α^2 's are proportional to σ_e , the 2×2 unit matrix, by Eqs. (11). Then, the change in energy is

$$\delta E' = -\frac{3}{4m\omega_A^2} J'^2_A - \frac{3J'^2_E}{m\omega_E^2}. \quad (25)$$

Note that the degeneracy is not lifted although the energy is decreased by distortions. The contribution of the A_1 mode to $\delta E'$ obtained here is equal with that by the static model, and the contribution of the E mode is twice that by the static model.

In the subspace of $|E\eta>_0$'s,

$$Q_A = -\sqrt{\frac{3}{2}} \frac{J'_A}{m\omega_A^2} \sigma_e, \quad (26)$$

$$Q_u = -\frac{\sqrt{3}J'_E}{m\omega_E^2} \sigma_z, \quad Q_v = -\frac{\sqrt{3}J'_E}{m\omega_E^2} \sigma_x. \quad (27)$$

The distortion due to the A_1 mode in $|E\eta>_0$ is

$$\langle E\eta|Q_A|E\eta>_0 = -\sqrt{\frac{3}{2}} \frac{J'_A}{m\omega_A^2} \quad (28)$$

for $\eta = u$ and v , and those due to the E_u mode for $|Eu>_0$ and $|Ev>_0$ states are

$$\langle Eu|Q_u|Eu>_0 = -\frac{\sqrt{3}J'_E}{m\omega_E^2}, \quad (29a)$$

$$\langle Ev|Q_u|Ev>_0 = +\frac{\sqrt{3}J'_E}{m\omega_E^2}, \quad (29b)$$

which have opposite sign according to the spin-states $|E\eta>_0$ with $\eta = u$ or v . In contrast to Q_A and Q_u , Q_v is non-diagonal and its expectation values are vanishing:

$$\langle E\eta|Q_v|E\eta>_0 = 0, \quad (30)$$

i.e., the distortion due to E_v mode is smeared out and can not be observed.

The fluctuation is estimated as

$$\langle E\eta|Q_v^2|E\eta>_0 = \frac{3J_E'^2}{m^2\omega_E^4} + \frac{\hbar}{2m\omega_E} \quad (31)$$

for both $\eta = u$ and v . The term with \hbar comes from the zero-point motion, and the first term is equal to $\langle E\eta|Q_u|E\eta>_0^2$ by Eq. (29). Although the tetrahedron distorts into different shapes depending on the spin-states $|E\eta>_0$ with $\eta = u$ or v , the spin-states remain doubly degenerate.

The expectation value of the squared displacement of the E mode distortion is

$$\langle E\eta|Q_u^2 + Q_v^2|E\eta>_0 = \frac{6J_E'^2}{m^2\omega_E^4} + \frac{\hbar}{m\omega_E}. \quad (32)$$

On the other hand, the sum of squared expectation values is

$$\langle E\eta|Q_u|E\eta>_0^2 + \langle E\eta|Q_v|E\eta>_0^2 = \frac{3J_E'^2}{m^2\omega_E^4}, \quad (33)$$

which is half of the first term in Eq. (32) apart from the contribution of the zero-point motion and equal to the value obtained by the static model, Eq. (15b). The change in energy due to the E mode distortion obtained

by dynamical model is twice that obtained by static model because of the quantum fluctuation.

Expanding J up to the quadratic terms of Q_α 's, we obtain the coupling with f_A for the A_1 representation as

$$\mathcal{H}''[A_1] = -[J_{AA}'' Q_A^2 + \frac{J_{AE}''}{\sqrt{2}} (Q_u^2 + Q_v^2) + \frac{J_{AT}''}{\sqrt{3}} (Q_1^2 + Q_2^2 + Q_3^2)] f_A, \quad (34)$$

with f_1 , f_2 and f_3 for the T_2 representation as

$$\begin{aligned} \mathcal{H}''[T_2] = & -\sqrt{2}J_{TA}'' (Q_A Q_1 f_1 + Q_A Q_2 f_2 + Q_A Q_3 f_3) \\ & -\frac{J_{TE}''}{2\sqrt{2}} \{ [Q_u Q_1 + Q_1 Q_u - \sqrt{3}(Q_v Q_1 + Q_1 Q_v)] f_1 \\ & + [Q_u Q_2 + Q_2 Q_u + \sqrt{3}(Q_v Q_2 + Q_2 Q_v)] f_2 \\ & + 2(Q_u Q_3 + Q_3 Q_u) f_3 \} \\ & -\frac{J_{TT}''}{\sqrt{2}} [(Q_2 Q_3 + Q_3 Q_2) f_1 + (Q_3 Q_1 + Q_1 Q_3) f_2 \\ & + (Q_1 Q_2 + Q_2 Q_1) f_3], \end{aligned} \quad (35)$$

and with f_u and f_v for the E representation as

$$\begin{aligned} \mathcal{H}''[E] = & -\sqrt{2}J_{EA}'' (Q_A Q_u f_u + Q_A Q_v f_v) \\ & -\frac{J_{EE}''}{\sqrt{2}} [(Q_u^2 - Q_v^2) f_u + (Q_u Q_v + Q_v Q_u) f_v] \\ & -J_{ET}'' \left(\frac{Q_1^2 + Q_2^2 - 2Q_3^2}{\sqrt{6}} f_u + \frac{Q_1^2 - Q_2^2}{\sqrt{2}} f_v \right). \end{aligned} \quad (36)$$

When J is a function of only the distance between spins,

$$\begin{aligned} J_{AA}'' &= 2\sqrt{6}J''/3, \quad J_{AE}'' = J''/\sqrt{3}, \\ J_{AT}'' &= 3/\sqrt{2} \left(J'/a + 2J''/3 \right), \\ J_{EA}'' &= -2/\sqrt{3}J'', \quad J_{EE}'' = -J''/\sqrt{6}, \quad J_{ET}'' = \sqrt{2}J'', \end{aligned} \quad (37)$$

where $J'' = \partial^2 J / \partial a^2$.

In $\mathcal{H}''[A_1]$, $Q_u^2 + Q_v^2$ is proportional to σ_e by Eq. (27) and Q_τ^2 's vanish apart from the zero-point motion by Eqs. (11c) and (23). In $\mathcal{H}''[E]$, $Q_u^2 - Q_v^2$ and $Q_u Q_v + Q_v Q_u$ vanish by Eq. (27). The last term vanishes by Eqs. (11c) and (23). Thus, \mathcal{H}'' is proportional to σ_e in the subspace of $|E\eta, S_z>_0$'s. Note that if Q_η is considered classical, $Q_u Q_v + Q_v Q_u$ does not vanish, so the degeneracy is lifted by f_v . After straightforward calculations, we obtain

$$\begin{aligned} \delta E'' = & \left[\frac{3\sqrt{6}}{8} \frac{J_A'^2}{m^2\omega_A^4} + \frac{\sqrt{3}\hbar}{4m\omega_A} \right] J_{AA}'' + \frac{3\sqrt{2}\hbar}{8m\omega_T} J_{AT}'' \\ & + \left[\frac{3\sqrt{3}}{2} \frac{J_E'^2}{m^2\omega_E^4} + \frac{\sqrt{3}\hbar}{4m\omega_E} \right] J_{AE}'' + \frac{3\sqrt{3}}{2} \frac{J_A'^2 J_E'^2 J_{EA}''}{(m\omega_A \omega_E)^4}. \end{aligned} \quad (38)$$

Now let us consider about a new mechanism for the tetragonal distortion at the phase transition without AF ordering in vanadium and nickel spinels with

spin-1.^{12,13,14} Yamashita and Ueda⁹ have considered the mechanism by breaking up the tetrahedron of spin-1 composing pyrochlore structure into the tetramer of spin-1/2 and by taking into account the spin-lattice coupling by the static model. They have explained the mechanism as a result of splitting of the degeneracy of the ground spin-state due to distortion of the cluster by considering higher terms more than quadratic terms to settle the ratio of Q_u and Q_v .

On the basis of the present dynamical model, the distortion of Q_u component emerges because the distortion of Q_v component is smeared out by the quantum-mechanical fluctuation. According to our model, the tetragonal distortion at the structural phase transition implies the occurrence of hidden ordering of the nonmagnetic spin-state $|\text{Eu}\rangle_0$, $\eta = u$ or v , because the sign of $\langle E\eta|Q_u|E\eta\rangle_0$ for $\eta = u$ or v is opposite to each other.

VI. DISCUSSION AND CONCLUSIONS

We have studied the frustrating quantum spin-1/2 system on the regular tetrahedron by taking into account the effect of distortion up to the quadratic terms. When the distortion is represented by the classical variables, the degeneracy of the ground spin-state is lifted by the distance dependence of exchange parameter. The energy of separated ground spin-state is determined through $Q_u^2 + Q_v^2$,

hence the ratio of Q_u and Q_v is left arbitrary, which gives continuous degeneracy with respect to the shape of the tetrahedron for the separated spin-state.

When the distortion is represented by the quantum-mechanical variables, the degeneracy of the modified spin-states $|\text{Eu}\rangle_0$ and $|\text{Ev}\rangle_0$ is not lifted by the magneto-elastic coupling although the cluster is distorted. The change in energy due to the distortion obtained by dynamical model is twice that obtained by the static model.

The distortions by Q_A (A_1 mode) and Q_u (E mode) do not fluctuate and their expectation values are equal to the results obtained by the static model apart from the contribution of the zero point motion. On the other hand, Q_v (E mode) is smeared out by the quantum fluctuation and does not contribute to the deformation in appearance. Then the shape of the distorted tetrahedron is determined by Q_u . The distortion of the tetrahedron is attributed to the quantum fluctuation caused by magneto-elastic coupling.

We have proposed a new model for the structural phase transition without AF ordering in vanadium and nickel spinels.^{12,13,14} The tetragonal distortion is caused by quantum fluctuation in contrast to the spin Jahn-Teller effect by Yamashita and Ueda.⁹ According to our model, the tetragonal distortion at the structural phase transition implies the occurrence of hidden ordering of the nonmagnetic spin-states $|\text{Eu}\rangle_0$ or $|\text{Ev}\rangle_0$.

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